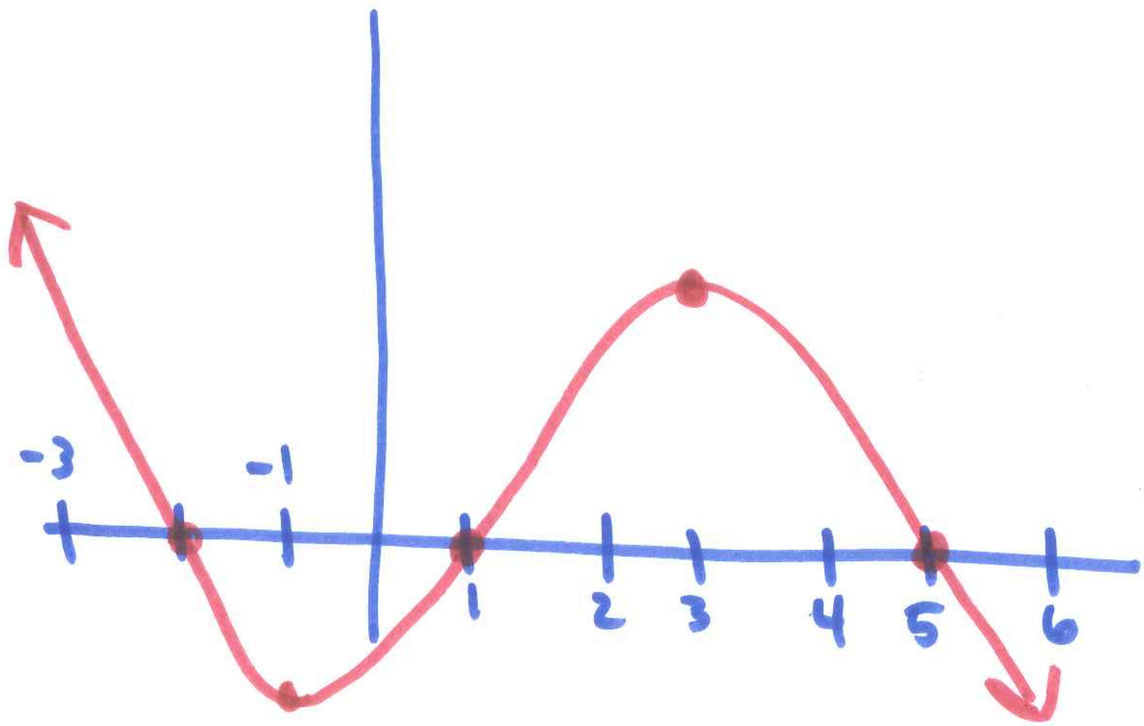
Chapter 6 - 7B Review

Ex: Consider the graph of $f'(x)$.



a) Where is $f(x)$ increasing?

$f'(x)$ is positive

$(-\infty, -2) \cup (1, 5)$

b) Where is $f(x)$ decreasing?

$f'(x)$ is negative

$(-2, 1) \cup (5, \infty)$

c) Where is $f(x)$ concave up?

$f''(x)$ is positive - Slope of tangent line to $f'(x)$ is positive

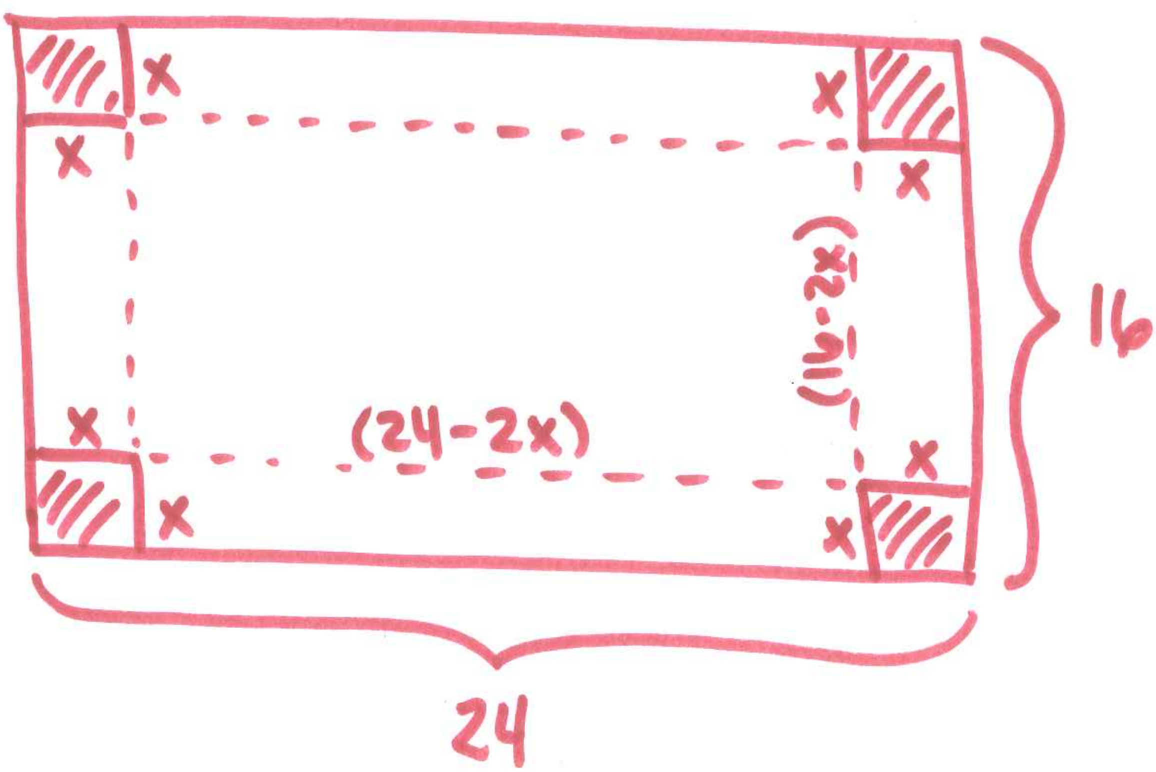
$(-1, 3)$

d) Where is $f(x)$ concave down?

$f''(x)$ is negative - Slope of tangent line of $f'(x)$ is negative.

$(-\infty, -1) \cup (3, \infty)$

<u>Ex</u>: An open box is to be made out of a 16×24 inch piece of cardboard by cutting out squares of equal size from the four corners and bending up the sides. Find the dimensions of the box with the largest volume.



We need to maximize Volume.

$$V = l \cdot w \cdot h$$
$$= (24-2x)(16-2x)(x)$$

distances are ≥ 0....

$l \geq 0$	$w \geq 0$	$h \geq 0$
$24-2x \geq 0$	$16-2x \geq 0$	$x \geq 0$
$24 \geq 2x$	$16 \geq 2x$	
$12 \geq x$	$8 \geq x$	

so $x \in [0,8]$

next step... take the derivative!

let $V(x) = (\underline{24-2x})(\underline{16-2x})(x)$
$$= (384 - 80x + 4x^2)(x)$$
$$= 384x - 80x^2 + 4x^3$$

then $V'(x) = 384 - 160x + 12x^2$

$$= 12x^2 - 160x + 384$$

$$= 4(3x^2 - 40x + 96)$$

$V'(x) = 0$ when $3x^2 - 40x + 96 = 0$

so $x = \dfrac{40 \pm \sqrt{40^2 - 4(3)(96)}}{2(3)}$

$$x = \frac{40 \pm \sqrt{448}}{6}$$

$$x = \frac{40 \pm 8\sqrt{7}}{6} = \frac{20 \pm 4\sqrt{7}}{3}$$

$x \approx \underbrace{10.194}$ and 3.139

$\dfrac{20 + 4\sqrt{7}}{3}$ is not in interval $[0, 8]$

V(x) is continuous on a closed and bounded interval... use EVT!

$V(0) = (+)(+)(0) = 0$

$V\left(\frac{20-4\sqrt{7}}{3}\right) = (+)(+)(+) = \text{"}+\text{"}$ ← max

$V(8) = (+)(0)(+) = 0$

Max volume occurs when

$x = \frac{20-4\sqrt{7}}{3} \approx \boxed{3.139 \text{ in} = h}$

then $\ell = 24 - 2x \approx \boxed{17.722 \text{ in} = \ell}$

and $w = 16 - 2x \approx \boxed{9.722 \text{ in} = w}$